MATHEMATICAL CHALLENGES

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1

THE MATHEMATICS OF THE BRAIN

Develop a mathematical theory to build a functional model of the brain that is mathematically consistent and predictive rather than merely biologically inspired.

2

THE DYNAMICS OF NETWORKS

Develop the high-dimensional mathematics needed to accurately model and predict behavior in large-scale distributed networks that evolve over time occurring in communication, biology, and the social sciences.

3

CAPTURE AND HARNESS STOCHASTICITY IN NATURE

Address Mumford's call for new mathematics for the 21ST century.
Develop methods that capture persistence in stochastic environments.

4

21ST CENTURY FLUIDS

Classical fluid dynamics and the Navier-Stokes Equation were extraordinarily successful in obtaining quantitative understanding of shock waves, turbulence, and solitons, but new methods are needed to tackle complex fluids such as foams, suspensions, gels, and liquid crystals.

5

BIOLOGICAL QUANTUM FIELD THEORY

Quantum and statistical methods have had great success modeling virus evolution. Can such techniques be used to model more complex systems such as bacteria? Can these techniques be used to control pathogen evolution?

6

COMPUTATIONAL DUALITY

Duality in mathematics has been a profound tool for theoretical understanding. Can it be extended to develop principled computational techniques where duality and geometry are the basis for novel algorithms?

7

OCCAM'S RAZOR IN MANY DIMENSIONS

As data collection increases can we "do more with less" by finding lower bounds for sensing complexity in systems? This is related to questions about entropy maximization algorithms.

8

BEYOND CONVEX OPTIMIZATION

Can linear algebra be replaced by algebraic geometry in a systematic way?

9

WHAT ARE THE PHYSICAL CONSEQUENCES OF PERELMAN'S PROOF OF THURSTON'S GEOMETRIZATION THEOREM?

Can profound theoretical advances in understanding three-dimensions be applied to construct and manipulate structures across scales to fabricate novel materials?

10

ALGORITHMIC ORIGAMI AND BIOLOGY

Build a stronger mathematical theory for isometric and rigid embedding that can give insight into protein folding.

11

OPTIMAL NANOSTRUCTURES

Develop new mathematics for constructing optimal globally symmetric structures by following simple local rules via the process of nanoscale self-assembly.

12

THE MATHEMATICS OF QUANTUM COMPUTING, ALGORITHMS, AND ENTANGLEMENT

In the last century we learned how quantum phenomena shape our world. In the coming century we need to develop the mathematics required to control the quantum world.

13

CREATING A GAME THEORY THAT SCALES

What new scalable mathematics is needed to replace the traditional PDE approach to differential games?

14

AN INFORMATION THEORY FOR VIRUS EVOLUTION

Why not?

15

THE GEOMETRY OF GENOME SPACE

What notion of distance is needed to incorporate biological utility?

16

WHAT ARE THE SYMMETRIES AND ACTION PRINCIPLES FOR BIOLOGY?

Extend our understanding of symmetries and action principles in biology along the lines of classical thermodynamics, to include important biological concepts such as robustness, modularity, evolvability, and variability.

17

GEOMETRIC LANGLANDS AND QUANTUM PHYSICS

How does the Langlands program, which originated in number theory and representation theory, explain the fundamental symmetries of physics? And vice versa?

18

ARITHMETIC LANGLANDS, TOPOLOGY, AND GEOMETRY

What is the role of homotopy theory in the classical, geometric, and quantum Langlands programs?

19

SETTLE THE RIEMANN HYPOTHESIS

The Holy Grail of number theory.

20

COMPUTATION AT SCALE

How can we develop asymptotics for a world with massively many degrees of freedom?

21

SETTLE THE HODGE CONJECTURE

This conjecture in algebraic geometry is a metaphor for transforming transcendental computations into algebraic ones.

22

SETTLE THE SMOOTH POINCARE CONJECTURE IN DIMENSION 4

What are the implications for space-time and cosmology? And might the answer unlock the secret of "dark energy"?

23

WHAT ARE THE FUNDAMENTAL LAWS OF BIOLOGY?

Dr. Tether's question will remain front and center in the next 100 years. I place this challenge last as finding these laws will undoubtedly require the mathematics developed in answering several of the questions listed above.

I am fully responsible for the selection
and statement of these challenges.

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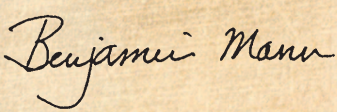

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